



Logic, Automata, and Games in Linear Temporal Logics on Finite Traces

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DECLARATIVE TO PROCEDURAL







Procedural Specs



Courtesy of Marco Montali

Giuseppe De Giacomo

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Procedural Specs Induce Traces







Traces Allowed by the Specs

A possibly infinite set of finite traces







Traces Allowed by the Specs







Traces Allowed by the Specs

Generalisation







Reality Is Often More Flexible Than It Seems



Courtesy of Marco Montali

Giuseppe De Giacomo





Declarative (Specifications of) Processes

Our goal







Declarative Processes

In late 2000's the Business Process Management (BPM) community produced a brilliant idea: [PesicVanDerAalst06] [AlbertiEtAlt06] [Montali2010] [PesicBovsnavkiVanDerAalst10]

The idea:

- Give the rules that a process should satisfy
- And nothing else!
- Extract the process from the rules only

Which specs?

- The one most used in formal methods for specifying process properties
- Linear Time Logic on Finite Traces (LTLf)

In other words:

- Drop explicit representation of process, and
- Use instead LTL formulas to specify the allowed process traces







Declarative Processes

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The idea:

- Give the rules that a process should satisfy
- And nothing else!
- Extract the process from the rules only

In other words: Automatically synthesize process from declarative specs

It can be seen as the fulfillment of the CS dream:

 Devise a technique for the "mechanical translation of human-understandable task specifications to a program that is known to meet the specifications." [Vardi - The Siren Song of Temporal Synthesis 2018] (more later)







Declarative Processes

Originally a controlled set of notable LTL formulas on were proposed for process specification (and a suitable graphical notation provided) [PesicVanDerAalst06] Can we use any LTL formula

Example (Main DECLARE Patterns)				as a declarative spec?
NAME	NOTATION	LTL_f	DESCRIPTION	
Existence	1* a	$\diamond a$	a must be executed at least once	No! What if the spec is:
Resp. existence	a • b	$\Diamond a \supset \Diamond b$	If a is executed, then b must be executed as well	always eventually Happy ?
Response	a 🔸 b	$\Box(a\supset\diamondsuit b)$	Every time a is executed, b must be executed afterwards	
Precedence	a 🔶 b	$ eg b \mathcal{W} a$	b can be executed only if a has been executed before	
Alt. Response	a ←→ b □($(a \supset \bigcirc (\neg a \mathcal{U} b))$	Every a must be followed by b, without any other a in Yes, if you focus on finite trace!	
Chain Response	a 🗪 b	$\square(a\supset \bigcirc b)$	If a is executed then b must be executed next (Specs in LILt instead of LIL)	
Chain Precedence	e a 🗪 b	$\Box(\bigcirc b \supset a)$	Task b can be executed only immediately after a	
Not Coexistence	a • III • b	$ eg(\diamondsuit a \land \diamondsuit b)$	Only one among tasks a and b can be executed	
Neg. Succession	a ● ⊪● b	$\Box(a \supset \neg \diamondsuit b)$	Task a cannot be followed by b, and b cannot be preceded by a	
Neg. Chain Succ.	a ●₩●● b	$\Box(a\supset \bigcirc \neg b)$	Tasks a and b cannot be executed next to each other	
Assumes only one activity (proposition) true at each Traces: Insensitivity to Infiniteness. AAAI 2014				





FOCUS ON FINITE TRACES

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Logic, Automata, and Games in Linear Temporal Logics on Finite Traces





Formal Methods

- Rigorous guarantees about the behavior of computational systems
- Wide-spread industrial adoption
- Main tasks
 - Formalisms for specs of dynamic properties:
 - E.g., Linear Temporal Logic
 - Verification:
 - Check if the system satisfies specs.
 - Synthesis:
 - Synthesize a system that satisfies specs.







Model Checking vs Synthesis

The "big bang" of the application of temporal logic to program verification: **Linear Temporal Logic** (LTL) (Pnueli, 1977)

Semantics of LTL is over ω -words, i.e., infinite traces

Note:

- ω -automata algorithms scale well as long as you do not have to determinize (\cdot)
- For synthesizing strategies/policy determinization is essential (•••
- In AI are more interested in finite-trace semantics





Focus on Finite Traces is Shared by Al

Planning in Al:

- Is all about having a task specification or "goal" and producing a "plan" (or strategy or policy) to satisfy the task in the environment model.
- Which tasks?
 - A task that terminates!
 - Typically, just reaching a certain state in the environment

Why tasks that terminates?

- Because it is the agent that is planning/reasoning
- If the task would not terminate, the agent would be stuck into doing the same task forever
- But then, why bother with equipping it with a model of the environment and of the task at all?
- Note it is the agent, NOT the designer, who has such a model

- In Formal Methods focus on infinite traces
- In AI focus on finite traces LTL → LTLf





Linear Time Temporal Logics on Finite Traces

 LTL_f/LDL_f : linear temporal logics on finite traces [DeGiacomoVardi2013] LTL_f : linear time temporal logic on finite traces Same syntax as standard LTL but interpreted over finite traces $\varphi ::= A \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid next \varphi \mid eventually \varphi \mid always \varphi \mid \varphi_1 until \varphi_2$ Examples: eventually A "eventually A" reachability "always A" always A safety "always if A then eventually B" $always(A \rightarrow eventually B)$ reactiveness "A until B" A until B until $\neg B \text{ until } A \lor always \neg B$ "A before B" precedence

 LDL_f : linear dynamic logic on finite traces

Same syntax as $\ensuremath{\operatorname{PDL}}$ but interpreted over finite traces

 $\varphi ::= tt \mid A \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \langle \rho \rangle \varphi \mid [\rho] \varphi \qquad \rho ::= A \mid \varphi? \mid \rho_1 + \rho_2 \mid \rho_1; \rho_2 \mid \rho^*$

Adds the possibility of expressing procedural constraints/goals [Reiter01], [BaierFritzMcllraith07]:

 $\delta ::= A \mid \varphi? \mid \delta_1 + \delta_2 \mid \delta_1; \delta_2 \mid \delta^* \mid \text{if } \phi \text{ then } \delta_1 \text{ else } \delta_2 \mid \text{while } \phi \text{ do } \delta$

where if and while are abbreviations: if ϕ then δ_1 else $\delta_2 \doteq (\phi?; \delta_1) + (\neg \phi?; \delta_2)$ and while ϕ do $\delta \doteq (\phi?; \delta)^*; \neg \phi?$

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Linear Time Temporal Logics on Finite Traces

Example

• "All coffee requests from person p will eventually be served":

 $\textit{always}(request_p \rightarrow \textit{eventually } coffee_p) \qquad [\texttt{true}^*](request_p \rightarrow \langle \texttt{true}^* \rangle coffee_p)$

• "Every time the robot opens door d it closes it immediately after":

 $always(openDoor_d \rightarrow next closeDoor_d)$ $[true^*]([openDoor_d]closeDoor_d)$

• "Before entering restricted area a the robot must have permission for a":

 $\neg inArea_a \text{ until } getPerm_a \lor always \neg inArea_a \qquad \qquad \langle (\neg inArea_a)^* \rangle getPerm_a \lor [\texttt{true}^*] \neg inArea_a$

• "Each time the robot enters the restricted area a it must have a new permission for a":

 $\langle (\neg inArea_a^*; getPerm_a; \neg inArea_a^*; inArea_a; inArea_a^*)^*; \neg inArea_a^* \rangle end$

• "At every point, if it is hot then, if the air-conditioning system is off, turn it on, else don't turn it off":



LTLf to Automata



DFA are indeed machines and hence processes!

Key point

 LTL_f/LDL_f formulas can be translated into a finite-state automaton on finite words \mathcal{A}_{φ} such that:

 $t \models \varphi \text{ iff } t \in \mathcal{L}(\mathcal{A}_{\varphi})$

- in linear time if \mathcal{A}_{φ} is an Alternating Automata (AFW);
- in exponential time if A_{φ} is an Nondeterministic Finite-state Automaton (NFA);
- in double exponential time if \mathcal{A}_{φ} is an Deterministic Finite-state Automaton (DFA).

We can compile reasoning into automata based procedures!





Regular Automata





Key point

 LTL_f/LDL_f formulas can be translated into deterministic finite state automata (DFA).

$$t \models \varphi \text{ iff } t \in \mathcal{L}(A_{\varphi})$$

where A_{arphi} is the DFA arphi is translated into.

Example (Automata for some LTL_f/LDL_f formulas)

PPLTL is a variant of LTLf that looks at traces backward (from now toward the past)

PPLTL: Pure Past LTL

$\varphi ::= A \mid \neg \varphi \mid \varphi_1 \land \varphi_2 \mid \ominus \varphi \mid \varphi_1 \, \mathcal{S} \, \varphi_2$

- PPLTL has the same expressive power of LTLf.
- The DFA \mathcal{A}_{Φ} of a PPLTL formula Φ is worst-case single exponential (vs double-exponential for LTLf):
 - build AFA reading the trace backward (same AFA as for LTLf linear)
 - compute the DFA of the reverse language (single exponential)
- Best algorithm for systematic translation between LTLf and PPLTL is 3EXPTIME,...
- ... but in several significant cases is polynomial, e.g., all DECLARE patterns [GeattiMontaliRivkinArXiv2022].

Actually PPLTL formulas can be evaluated in a dynamic programming method using only two instants:

- Now
- One step in the past (prev)

Current and Previous Instants

PPLTL formulas can be evaluated considering only the current and previous instants:

 $\begin{array}{lll} eval(A, \Pi_{now}, \Pi_{prev}) &=& A \text{ if variable "}A" \text{ is true in } \Pi_{now} \\ eval(\ominus \varphi, \Pi_{now}, \Pi_{prev}) &=& \ominus \varphi \text{ if variable "}\ominus \varphi" \text{ is true in } \Pi_{prev} \\ eval(\neg \varphi, \Pi_{now}, \Pi_{prev}) &=& \neg eval(\varphi, \Pi_{now}, \Pi_{prev}) \\ eval(\varphi_1 \land \varphi_2, \Pi_{now}, \Pi_{prev}) &=& eval(\varphi_1, \Pi_{now}, \Pi_{prev}) \land eval(\varphi_2, \Pi_{now}, \Pi_{prev}) \\ eval(\varphi_1 \lor \varphi_2, \Pi_{now}, \Pi_{prev}) &=& eval(\varphi_1, \Pi_{now}, \Pi_{prev}) \land eval(\varphi_2, \Pi_{now}, \Pi_{prev}) \\ eval(\varphi_1 \And \varphi_2, \Pi_{now}, \Pi_{prev}) &=& eval(\varphi_1, \Pi_{now}, \Pi_{prev}) \land eval(\varphi_2, \Pi_{now}, \Pi_{prev}) \\ eval(\varphi_1 \And \varphi_2, \Pi_{now}, \Pi_{prev}) &=& eval([\varphi_2 \lor (\varphi_1 \land \ominus (\varphi_1 \And \varphi_2))], \Pi_{now}, \Pi_{prev}) \end{array}$

Remember that the fixpoint equation for $\varphi_1 \mathcal{S} \varphi_2$ is $\varphi_1 \mathcal{S} \varphi_2 \equiv \varphi_2 \lor (\varphi_1 \land \ominus(\varphi_1 \mathcal{S} \varphi_2)).$

As a result one can built a symbolic DFA in linear time!

Symbolic DFA

$$\mathcal{A}_{\Phi} = (AP, PP, \Pi^{0}_{prev}, Trans, Final)$$

where

- AP set of boolean variable, one for each atomic proposition, representing which propositions are true/false in the current instant;
- *PP* set of boolean variable, one component for each sub-formulas the form ⊖φ in Fisher-Ladner closure of Φ, representing which formulas were true/false at the previous instant;
- $\Pi^0_{prev} = (false, \dots, false)$, initially all formulas of the form $\ominus \varphi$ are false
- $Trans(\Pi_{now}, \Pi_{prev}) = Trans_i(\Pi_{now}, \Pi_{prev}) \times \cdots \times Trans_n(\Pi_{now}, \Pi_{prev})$ with $n = |\Pi_{prev}|$, lat where for each variable " $\ominus \varphi_i$ ":

 $Trans_i(\Pi_{now}, \Pi_{prev}) = eval(\varphi_i, \Pi_{now}, \Pi_{prev})$

• $Final(\Pi_{now}, \Pi_{prev}) \equiv eval(\Phi, \Pi_{now}, \Pi_{prev})$

Important: \mathcal{A}_{Φ} is linear in Φ .

Several Applications of LTLf/PPLTL Specs

Many Applications:

- Planning for temporally extended goals
- Several forms of Synthesis
- MDP with non-Markovian rewards
- Reinforcement Learning for non-Markovian tasks
- Declarative Process Specification in BPM

DECLARATIVE TO PROCEDURAL TO GAMES

Specify Models and Tasks as Processes in Formal Methods

- Environment Model (ENV)
 - Spec. of environment possible behaviors (ENV)
 - Think of each **behavior** as a choice function resolving nondeterminism of a nondeterministic domain
 - ENV expressed as
 - nondeterministic planning domains
 - LTL/LTLf specifications
- Agent's Task (TASK/GOAL)
 - Spec. of agent's task
 - TASK expressed in LTL/LTLf
- Find agent's plan/behavior/policy/strategy that fulfills TASK against all behaviors of ENV

Specify ENV and TASK as with formalism used in Formal Methods e.g., LTL/LTLf

Nondeterministic Domains as Deterministic Automata

Nondeterministic domain (including initial state)

 $\mathcal{D} = (2^\mathcal{F}, \mathcal{A}, s_0, \delta, lpha)$ where:

- *F* fluents (atomic propositions)
- *A* actions (atomic symbols)
- $2^{\mathcal{F}}$ set of states
- s_0 initial state (initial assignment to fluents)
- $\alpha(s) \subseteq \mathcal{A}$ represents action preconditions
- $\delta(s, a, s')$ with $a \in \alpha(s)$ represents action effects (including frame).

Automaton A_D for \mathcal{D} is a DFA!!!

 $A_{\mathcal{D}} = (2^{\mathcal{F} \cup \mathcal{A}}, (2^{\mathcal{F}} \cup \{s_{init}\}), s_{init}, \varrho, F)$ where:

- $2^{\mathcal{F} \cup \mathcal{A}}$ alphabet (actions \mathcal{A} include dummy *start* action)
- $2^{\mathcal{F}} \cup \{s_{init}\}$ set of states
- s_{init} dummy initial state
- $F = 2^{\mathcal{F}}$ (all states of the domain are final)

•
$$ho(s, [a, s']) = s'$$
 with $a \in lpha(s)$, and $\delta(s, a, s')$ $ho(s_{init}, [start, s_0]) = s_0$

(notation: [a,s'] stands for $\{a\}\cup s')$

Nondeterministic Domains as Deterministic Automata

Example (Simplified Yale shooting domain)

• Domain \mathcal{D} :

• DFA $A_{\mathcal{D}}$:

Planning in Nondeterministic Domains for Reachability Goals

Planning in nondeterministic domains

- Set the arena formed by all traces that satisfy both the DFA A_D for D and the DFA for $\diamond G$ where G is the goal.
- Compute a winning strategy.

(EXPTIME-complete in \mathcal{D} , constant in G)

Planning in Nondeterministic Domains for Arbitrary LTLf Goals

In general, we need first to determinize the NFA for LTL_f/LDL_f formula

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(DFA can be exponential in NFA in general)

Planning in Nondeterministic Domains for LTLf Goals

DFA games

A DFA game $\mathcal{G} = (2^{\mathcal{F} \cup \mathcal{A}}, S, s_{init}, \varrho, F)$, is such that:

- \mathcal{F} controlled by environment; \mathcal{A} controlled by agent;
- $2^{\mathcal{F}\cup\mathcal{A}}$, alphabet of game;
- S, states of game;
- s_{init} , initial state of game;
- $\varrho: S \times 2^{\mathcal{F} \cup \mathcal{A}} \to S$, transition function of the game: given current state s and a choice of action a and resulting fluents values E the resulting state of game is $\varrho(s, [a, E]) = s'$;
- F, final states of game, where game can be considered terminated.

Winning Strategy:

- A play is winning for the agent if such a play leads from the initial to a final state.
- A strategy for the agent is a function $f: (2^{\mathcal{F}})^* \to \mathcal{A}$ that, given a history of choices from the environment, decides which action \mathcal{A} to do next.
- A winning strategy is a strategy $f: (2^{\mathcal{F}})^* \to \mathcal{A}$ such that for all traces π with $a_i = f(\pi_{\mathcal{F}}|_i)$ we have that π leads to a final state of \mathcal{G} .

This is the **game area**, in which agent and env will play!

It **is not only the domain**, but it is obtained from the **domain** and the **DFA of the formula**

Planning in Nondeterministic Domains for LTLf Goals

Winning states for DFA games

Let denote the set of final states of ${\mathcal G}$ as:

$$\llbracket F \rrbracket = \{ s \in \mathcal{S} \mid s \models F \}$$

and let's define the (adversarial preimage of a set \mathcal{E} the following function:

```
PreAdv(\mathcal{E}) = \{s \in \mathcal{S} \mid \exists a \in \alpha(s). \forall s' \in \mathcal{S}. \, \delta(s, a, s') \supset s' \in \mathcal{E}\}
```

Compute the set *Win* of winning states of DFA game, i.e., states from which the agent can reach the final states *F*, by least-fixpoint:

- $Win_0 = \llbracket F \rrbracket$ (the final states)
- $Win_{i+1} = Win_i \cup PreAdv(Win_i)$
- $Win = \bigcup_i Win_i$

(Computing Win is linear in the number of states in \mathcal{G})

Computing the winning strategy

Let's define $\omega: S \to 2^{\mathcal{A}}$ as: $\omega(s) = \{a \in \alpha(s) \mid \text{ if } s \in Win_{i+1} - Win_i \text{ then } \forall s'.\delta(s, a, s') \supset s' \in Win_i\}$

- Every way of restricting $\omega(s)$ to return only one action (chosen arbitrarily) gives a winning strategy for \mathcal{G} .
- Note s is a state of the game! not of the domain only!
 To phrase ω wrt the domain only, we need to return a stateful transducer with transitions from the game.

Planning in Nondeterministic Domains for LTLf Goals

Planning in Nondeterministic Domains for PPLTL Goals

(c.f. data vs query complexity in Databases)

Planning in Nondeterministic Domains for PPLTL Goals

Note we have separated costs in the model (DOM) and the task GOAL! (c.f. data vs query complexity in Databases)

DECLARATIVE CONTROL IN RL

DECLARATIVE

... TO GAMES

... TO PROCEDURAL

Combining Learning and Reasoning

Merging:

- Learning agent
 - Does reinforcement learning
 - Possibly deep reinforcement learning
- Reasoning agent
 - Does reasoning
 - Possibly on temporal specification as in formal methods

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Reasoning Agent (Restraining Bolt)

Learning Agent

Environment

rewards from reasoning

action

Action

Actuator

Restraining Bolts as Reasoning Agents

Example: Breakout

Learning Agent

- Features: paddle position, ball speed/position
- Actions: move the paddle
- Rewards: reward when a brick is hit

Restraining Bolt (Reasoning Agent)

- Rewards: break one column at the time left to right (all bricks in column i must be removed before completing any other column j > i)
- Fluents: bricks/columns status (broken/not broken)

The Agent Ignores the Fluents!

How the world is

How the agent sees the world

Controlling RL with Logic for Safety

The idea of restraining bolt can be subscribed to that part of research generated by the urgency of providing **safety guarantees** to AI techniques based on learning.

- S. Russell, D. Dewey, and M. Tegmark. Research priorities for robust and beneficial artificial intelligence. Al Magazine, 36(4), 2015.
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- D. Amodei, C. Olah, J. Steinhardt, P. Christiano, J. Schulman, and D. Mane. Concrete problems in Al safety. CoRR, abs/1606.06565, 2016.
- Mohammed Alshiekh, Roderick Bloem, Rüdiger Ehlers, Bettina Konighofer, Scott Niekum, Ufuk Topcu: Safe Reinforcement Learning via Shielding. AAAI 2018.
- Min Wen, Rüdiger Ehlers, Ufuk Topcu: Correct-by-synthesis reinforcement learning with temporal logic constraints IROS 2015.

Human Compatible

"The most important book I have read in qui

ARTIFICIAL INTELLIGENCE AND THE PROBLEM OF CONTROL

Stuart Russell

Shields

Poss(a,h) iff $h \models \varphi_{\text{LTLf}}$

Restraining Bolts as Shields for Frame

