Best-effort synthesis is a suitable form of planning, finds a strategy that ensures the agent will do its best to achieve the goal, i.e., a best-effort strategy.

\[ \text{LTL}_f \text{ best-effort synthesis} \] both the environment assumption and the agent goal are expressed as \( \text{LTL}_f \) formulas.
LTL<sub>f</sub> Reactive Synthesis Under Environment Assumptions

- **Reactive synthesis**, a *general form of planning*, finds an agent strategy that achieves the given goal (temporal goal)

- An agent strategy is a function $\sigma_{ag} : (2^X)^+ \rightarrow 2^Y$

---

**Given**: Environment assumption $\mathcal{E}$, agent goal $\varphi$, LTL<sub>f</sub> formulas over $X \cup Y$

**Obtain**: An agent strategy $\sigma_{ag}$ such that

$$\forall \sigma_{env} \triangleright \mathcal{E}, \pi(\sigma_{ag}, \sigma_{env}) \models \varphi$$
Best-effort synthesis finds a best-effort strategy, i.e., a strategy that ensures the agent does its best to achieve the goal.

## Dominance

Let $\sigma_1$ and $\sigma_2$ be two agent strategies. $\sigma_1$ dominates $\sigma_2$ for goal $\varphi$ under assumption $E$, written $\geq_{\varphi|E}$, if for every $\sigma_{\text{env}} \triangleright E, \pi(\sigma_2, \sigma_{\text{env}}) \models \varphi$ implies $\pi(\sigma_1, \sigma_{\text{env}}) \models \varphi$. $\sigma_1$ strictly dominates $\sigma_2$, written $\sigma_1 >_{\varphi|E} \sigma_2$, if $\sigma_1 \geq_{\varphi|E} \sigma_2$ and $\sigma_2 \not\geq_{\varphi|E} \sigma_1$.

## LTL$_f$ Best-Effort Synthesis Under Environment Assumptions

**Given:** Environment assumption $E$, agent goal $\varphi$, LTL$_f$ formulas over $X \cup Y$

**Obtain:** An agent strategy $\sigma$ such that there is no strategy $\sigma'$ that strictly dominates $\sigma$.
Contributions

▪ Study of the relationship between reactive synthesis and best-effort synthesis for specifications in Linear Temporal Logic on Finite Traces ($LTL_f$)

▪ Three novel symbolic approaches to $LTL_f$ best-effort synthesis:
  ▪ Monolithic
  ▪ Explicit-compositional
  ▪ Symbolic-compositional

▪ Empirical evaluation
Contributions

- Study of the relationship between reactive synthesis and best-effort synthesis for specifications in Linear Temporal Logic on Finite Traces ($\text{LTL}_f$)

- Three novel symbolic approaches to $\text{LTL}_f$ best-effort synthesis:
  - Monolithic
  - Explicit-compositional
  - Symbolic-compositional

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Contributions

- Study of the relationship between reactive synthesis and best-effort synthesis for specifications in Linear Temporal Logic on Finite Traces (LTL$_f$)

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  - Monolithic
  - Explicit-compositional
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- Empirical evaluation
Symbolic DFA Games

- The proposed approaches are based on a reduction to solving adversarial/cooperative reachability games on symbolic DFAs

Symbolic DFA [Zhu et al. 2017]

The symbolic representation of a DFA is a tuple \( G^s = (X, Y, Z, Z_0, \eta, f) \) where:

- \( X \) and \( Y \) are environment and agent variables, respectively
- \( Z \) is the set of state variables
- \( Z_0 \) is the initial state
- \( \eta : 2^X \times 2^Y \times 2^Z \rightarrow 2^Z \) represents the transitions of the DFA game
- \( f \) represents the final state of the DFA game
Symbolic DFA Games
Computing Adversarially Winning and Cooperatively Winning Strategies

▪ Winning strategy of an adversarial reachability game. Least fixpoint computation on Boolean formulas \( w \) and \( t \):

\[
\begin{align*}
t_{i+1}(Z, Y, Y) &= t_i(Z, X, Y) \lor (\neg w_i(Z) \land w_i(\eta(X, Y, Z))) \\
w_{i+1}(Z) &= \forall X. \exists Y. t_{i+1}(Z, X, Y);
\end{align*}
\]

▪ Winning strategy of a cooperative reachability game. Least fixpoint computation on Boolean formulas \( \hat{w} \) and \( \hat{t} \):

\[
\begin{align*}
\hat{t}_{i+1}(Z, Y, Y) &= \hat{t}_i(Z, X, Y) \lor (\neg \hat{w}_i(Z) \land \hat{w}_i(\eta(X, Y, Z))) \\
\hat{w}_{i+1}(Z) &= \exists X. \exists Y. \hat{t}_{i+1}(Z, X, Y);
\end{align*}
\]

▪ Fixpoint reached when \( w_{i+1} \equiv w_i \) (resp. \( \hat{w}_{i+1} = \hat{w}_i \))

▪ Computation of positional strategy by Boolean synthesis
Monolithic Approach

Figure: Monolithic

Figure: Explicit-Compositional

Figure: Symbolic-Compositional
Explicit-Compositional Approach

Figure: Monolithic

Figure: Explicit-Compositional

Figure: Symbolic-Compositional
Explicit-Compositional Approach

- Monolithic approach
  - LTLf to Explicit State DFA (Minimal)
    - Symbolic DFA
      - Symbolic Games Construction via Symbolic Product
        - Adv. DFA Game
          - τ_{ag}
        - Coop. DFA Game
          - W_{out}(D^s, f_{E ∧ ϕ})
  - Strategy Merging
    - Best-Effort Strategy

- Explicit-Compositional approach
  - LTLf to Explicit State DFA (Minimal)
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Implementation of the symbolic approaches in a tool called BeSyft:

- Monolithic-BeSyft
- Explicit-compositional-BeSyft
- Symbolic-compositional-BeSyft
Empirical Evaluation

- **Experiments** performed on a scalable benchmark **counter games**:
  - Performance comparison of the three symbolic approaches
  - Performance comparison of best-effort and reactive synthesis
  - Evaluation of the bottleneck and impact of the cooperative phase
Experimental Results
Comparing Symbolic Approaches

![Graph showing time comparison of different approaches]

- Monolithic-BeSyft
- Explicit-Compositional BeSyft
- Symbolic-Compositional BeSyft

Time (ms)
Experimental Results
Comparing Best-Effort Synthesis and Reactive Synthesis

![Bar chart comparing Symbolic-Compositional BeSyft, Adversarial Synthesis, and Cooperative Synthesis over different scenarios.](image)
Experimental Results
Relative Time Cost Evaluation

- LTLf to DFA Transformation
- Symbolic DFA Construction
- Adversarial Game
- Restriction
- Cooperative Game
- Merging

Bars represent the time cost percentages for different techniques with varying number of bits. The x-axis shows the time in milliseconds, and the y-axis represents the number of bits.
Conclusion and Future Work

- Three symbolic approaches to $\text{LTL}_f$ best-effort synthesis
- The symbolic-compositional approach has the best performance
- Automata minimization does not always lead to improvement
- $\text{LTL}_f$-to-DFA conversion is the bottleneck of $\text{LTL}_f$ best-effort synthesis.
- Performing best-effort synthesis only brings minor overhead comparing with standard reactive synthesis

Future Directions

- $\text{LTL}_f$ best-effort synthesis on planning domains
- $\text{LTL}_f$ best-effort synthesis under multiple environment assumptions
- $\text{LTL}$ best-effort synthesis